

Selection rules and quark correlations in the N^* resonance spectrum

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Abstract. A “ Λ selection rule” for N^* resonances in the presence of QCD mixing effects is identified. Due to the QCD mixing, excitations of **20**-plets are possible in SU(6). We show that this selection rule is useful for classifying PDG states at $N = 2$, and for clarifying whether strongly correlated diquarks survive for $L > 0$.

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1 Quark model and selection rules

Although it has been about forty years since the quark model was first applied to the problem of baryon resonances, it is still not well established whether three constituent quarks or a quark-diquark effective degrees of freedom are needed in the description of the baryon spectrum. In the recent years, significant progresses on the photo-nuclear reactions as a probe for the internal structure of nucleon and nucleon resonance have been made in experiment, which provide not only stringent constraints on theoretical phenomenologies but also novel insights into the strong QCD dynamics in this challenging regime.

A standard and phenomenologically successful assumption common to a large number of papers in the quark model is that photon transitions are additive in the constituent quarks [1,2,3]. This assumption also underlies models of hadronic production and decay in the sense that when $q_1 q_2 q_3 \rightarrow [q_1 q_2 q_i] + [q_3 \bar{q}_i]$, the quark pair $q_1 q_2$ are effectively spectators and only q_3 is involved in driving the transition. Such approximations lead to well known selection rules, which have proved useful in classifying resonances[1]. We adopt this approximation as a first step and show that within it there is a further selection rule that appears to have been overlooked in the literature. We shall refer to this as the “ Λ selection rule” and show how it may help classify N^* resonances [4].

The standard SU(6) \otimes O(3) wavefunction can be constructed from three fundamental representations of group S_3 :

$$SU(6) : \mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_s + \mathbf{70}_p + \mathbf{70}_\lambda + \mathbf{20}_a, \quad (1)$$

where the subscripts denote the corresponding S_3 basis for each representation, and the bold numbers denote the

dimension of the corresponding representation. The spin-flavor wavefunctions can be expressed as $[\mathbf{N}_6, {}^{2S+1}\mathbf{N}_3]$, where \mathbf{N}_6 (**=56, 70 or 20**) and \mathbf{N}_3 (**=8, 10, or 1**) denote the SU(6) and SU(3) representation and S stands for the total spin. The SU(6) \otimes O(3) (symmetric) wavefunction is

$$[\text{SU}(6) \otimes \text{O}(3)] = [\mathbf{N}_6, {}^{2S+1}\mathbf{N}_3, N, L, J], \quad (2)$$

where explicit expressions follow the convention of Isgur and Karl [5,6,7].

The basic rules follow from application of the Pauli exclusion principle to baryon wavefunctions together with an empirically well tested assumption that electroweak and strong decays are dominated by single quark transitions where the remaining two quarks, or diquark, are passive spectators [8]. As a consequence, it leads to a correlated vanishing transition matrix element between N^* of **[70, 48]** and **[56, 28]** in $N^* \rightarrow \Lambda K$ or ΛK^* . This follows because the $[ud]$ in the Λ has $S = 0$ and in the spectator approximation, the strangeness emissions in $N^* \rightarrow \Lambda K$ or ΛK^* , the spectator $[ud]$ in the N^* must also be in $S_{[ud]} = 0$, whereby such transitions for the N^* of **[70, 48]** with $S_{[ud]} = 1$ are forbidden.

This “ Λ selection rule”, which appears to have been overlooked in the literature, seems to be useful for resolving the underlying transition dynamics and probing the structure of excited N^* . We note the well-known Moorhouse selection rule [9] which states that transition amplitudes for γp to all resonances of representation **[70, 48]**, such as $D_{15}(1675)$, must be zero due to the vanishing transition matrix element for the charge operator. In contrast, the Λ selection rule applies to both proton and neutron resonances of **[70, 48]**. We also note that $\Lambda^*[\mathbf{70}, {}^4 8] \rightarrow \bar{K} N$ has been discussed in Ref. [10,11]. But the source and gen-

erality of the Λ selection rule does not seem to have been noted [11].

2 Recognition of the Λ selection rule

An immediate application of the selection rules is to the $D_{15}(1675)$, which is in [70,⁴8]. According to the Moorhouse selection rule, the amplitudes for $\gamma p \rightarrow D_{15}$ should vanish. However, the experimental values are not zero, though they are small. Non-zero amplitudes arise from QCD mixings induced by single gluon exchange in the physical nucleon [12]. The effective interaction

$$H_{FB} = \frac{2\alpha_s}{3m_i m_j} \left[\frac{8\pi}{3} \mathbf{S}_i \cdot \mathbf{S}_j \delta^3(\mathbf{r}_{ij}) + \frac{1}{r_{ij}^3} \left(\frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^2} - \mathbf{S}_i \cdot \mathbf{S}_j \right) \right] \quad (3)$$

induces significant mixings between the $^2\mathbf{8}$ and $^4\mathbf{8}$ in the $\mathbf{56}$ and $\mathbf{70}$ [7] and the nucleon wavefunction becomes [12]

$$|N\rangle = 0.90|^2S_S\rangle - 0.34|^2S_{S'}\rangle - 0.27|^2S_M\rangle - 0.06|^4D_M\rangle, \quad (4)$$

where subscripts, S and M , refer to the spatial symmetry in the S and D -wave states for the nucleon internal wavefunction. Thus, the $O(\alpha_s)$ admixtures at $N = 2$ comprise a 34% in amplitude excited $\mathbf{56}$ and 27% $\mathbf{70}$ each with $L = 0$ and 6% $\mathbf{70}$ with $L = 2$. The following points can be learned by applying the rules to compare with the experimental observations:

i) Due to the QCD mixing in the wavefunction of the nucleon, the Moorhouse selection rule is violated. The $\mathbf{70}$ admixture quantitatively agrees with the most recent data [13] for the $\gamma p \rightarrow D_{15}$ amplitudes, neutron charge radius and $D_{05} \rightarrow \bar{K}N$ [12]. The results assume that mixing effects in the D_{15} are negligible relative to those for the nucleon [12]: this is because there is no $[\mathbf{70}, ^2\mathbf{8}; L^P = 1^-]$ state available for mixing with the D_{15} , and the nearest $J = 5/2$ state with negative parity is over 500 MeV more massive at $N = 3$. The leading $O(\alpha_s)$ amplitude for $\gamma p \rightarrow D_{15}$ is dominantly driven by the small components in the nucleon and the large component in the D_{15} [6] for which $\Delta N = 1$.

ii) The Λ selection rule remains robust, or at least as good as the Moorhouse rule even at $O(\alpha_s)$. This is because in the context of the diquark model, admixtures of $[ud]$ with spin one, which would violate the selection rule, are only expected at most to be 20% in amplitude [14], to be compared with 27% for the nucleon in Eq. (4). Thus decays such as $D_{15} \rightarrow K\Lambda$ will effectively still vanish relative to $K\Sigma$; for the $D_{15}(1675)$ the phase space inhibits a clean test but the ratio of branching ratios for the analogous state at $N = 2$, namely $F_{17}(1990) \rightarrow K\Lambda : K\Sigma$, may provide a measure of its validity.

iii) For $\gamma n \rightarrow D_{15}$, where the Moorhouse selection rule does not apply, the amplitudes are significantly large and consistent with experiment [13]. However, due to the Λ selection rule, the $D_{15}^0 \nrightarrow K^0\Lambda$ which makes the search

for the D_{15} signals in $\gamma N \rightarrow K\Lambda$ interesting. An upper limit of $B.R. < 1\%$ is set by the PDG [13] which in part may be due to the limited phase space; a measure of the ratio of branching ratios for $K\Lambda : K\Sigma$ would be useful. The $F_{17}(1990)$, which is the only F_{17} with $N = 2$, is an ideal candidate for such a test, which may be used in disentangling the assignments of the positive parity N^* at the $N = 2$ level.

3 Excitation of 20-plets

The QCD admixture of $[\mathbf{70}, ^2\mathbf{8}, 2, 0, 1/2]$ in the nucleon wavefunction enables the excitation of $\mathbf{20}$ -plets. There has been considerable discussion as to whether the attractive forces of QCD can cluster $[ud]$ in color $\bar{\mathbf{3}}$ so tightly as to make an effective bosonic “diquark” with mass comparable to that of an isolated quark. Comparison of masses of $N^*(u[ud])$ and mesons $u\bar{d}$ with $L \geq 1$ support this hypothesis of a tight correlation, at least for excited states [15]. If the quark-diquark dynamics is absolute, then $SU(6) \otimes O(3)$ multiplets such as $[\mathbf{20}, 1^+]$ cannot occur. The spatial wavefunction for $\mathbf{20}$ involves both ρ and λ degrees of freedom; but for an unexcited diquark, the ρ oscillator is frozen. Therefore, experimental evidence for the excitations of the $\mathbf{20}$ plets can distinguish between these prescriptions.

In Ref. [4] the transition amplitudes for the lowest $[\mathbf{20}, 1^+]$ to $\mathbf{70}$ -plets are given. It is also shown that the amplitudes are compatible with the Moorhouse-selection-rule-violating amplitudes in $\gamma p \rightarrow D_{15}$. Thus, a 27% $\mathbf{70}$ admixture in the nucleon has potential implications for resonance excitation that may be used to look for $\mathbf{20}$ -plets. Nevertheless, additional P_{11} and P_{13} from representation $\mathbf{20}$ automatically raise questions about the quark model assignments of the observed P_{11} and P_{13} states, among which $P_{11}(1440)$, $P_{11}(1710)$, and $P_{13}(1720)$ are well established resonances, while signals for $P_{13}(1900)$ and $P_{11}(2100)$ are quite poor [13].

4 Implications from the present experimental data

At $N = 2$ in the quark model a quark-diquark spectrum allows $[\mathbf{56}, 0^+]$, $[\mathbf{56}, 2^+]$, and $[\mathbf{70}, 0^+][\mathbf{70}, 2^+]$. If all qqq degrees of freedom can be excited, correlations corresponding to $[\mathbf{20}, 1^+]$ are also possible. As shown in Ref. [4], without $\mathbf{20}$ -plets, most of the observed states can fit in the $SU(6)$ scheme. However, there are still a lot of problems and controversies. For instance, neither of the $P_{13}(1710/1900)$ fit easily with being pure $\mathbf{56}$ or $\mathbf{70}$ states; states of $[\mathbf{70}, ^2\mathbf{8}, 2, 2, J]$ are still missing, and another P_{13} and F_{15} are needed.

When the QCD mixing effects are included the agreement improves, in that small couplings of 4N states to γp are predicted, in accord with data. However, the implication is the added complexity that an additional P_{11} and P_{13} correlation in $[\mathbf{20}, 1^+]$ is allowed. Most immediately this prevents associating the $P_{11}(1710)$ as $[\mathbf{70}, 0^+]$ simply

Table 1. Helicity amplitudes for the $P_{11}(1710)$ and $P_{13}(1720)$ with all the possible quark model assignments for them. The data are from PDG [13], and numbers have a unit of $10^3 \times \text{GeV}^{1/2}$.

	$P_{11}(1710)$ $A_{1/2}^p$	$P_{13}(1720)$ $A_{1/2}^p$ $A_{3/2}^p$		$P_{13}(1900)$ $A_{1/2}^p$ $A_{3/2}^p$	
$[56,^2 8; 2^+]$	*	100	30	110	39
$[70,^2 8; 0^+]$	32	*	*	*	*
$[70,^2 8; 2^+]$	*	-71	-21	-78	-28
$[70,^4 8; 0^+]$	*	17	29	14	24
$[70,^4 8; 2^+]$	-8	7	12	9	16
$[20,^2 8; 1^+]$	-15	-11	-18	-10	-17
Exp. data	9 ± 22	18 ± 30	-19 ± 20	-17	+31

on the grounds of elimination of alternative possibilities. Thus we now consider what are the theoretical signals and what does experiment currently say.

Qualitatively one anticipates $P_{13}(^4N)$ having a small but non-zero coupling to γp , the γn being larger while the $K\Lambda$ decay is still forbidden. For the **20** states $P_{11,13}(^2N)$ both γp and γn amplitudes will be small and of similar magnitude. However, mixing with their counterparts in **56** and **70** may be expected. In Table 1, we list the helicity amplitudes for the $P_{11}(1710)$, $P_{13}(1720)$ and $P_{13}(1900)$ with all the possible quark model assignments and the mixing angles from Eq. (4). The amplitudes for the P_{11} and P_{13} of $[20,^2 8, 2, 1, J]$ are the same order of magnitude as the Moorhouse-violating $\gamma p \rightarrow D_{15}(1675)$ [4]. For the $P_{11}(1710)$ all three possible configurations have amplitudes compatible with experimental data. For $P_{13}(1720)$, assignment in either $[56,^2 8; 2^+]$ or $[70,^2 8; 2^+]$ significantly overestimates the data [16, 17] for $A_{1/2}^p$ if it is a pure state.

Table 1 shows that the presence of **20** cannot be ignored, should be included in searches for so-called “missing resonances”, and that a possible mixture of the **20**-plets may lead to significant corrections to the results based on the conventional **56** and **70**. This raises a challenge for experiment, whether one can eliminate the *extreme* possibility that $P_{11}(1710)$ and $P_{13}(1720)$ are consistent with being in **20** configurations. There are already qualitative indications that they are not simply **56** or **70**. Their hadronic decays differ noticeably from their sibling $P_{11}(1440)$: compared with the $P_{11}(1440)$ in **56** for which $\Gamma_T \sim 350$ MeV with a strong coupling to $N\pi$, their total widths are ~ 100 MeV, with $N\pi$ forming only a small part of this.

Some further implications due to the **20**-plets excitations can be learned here:

i) For a state of **20**-plet, as $20 \rightarrow 56 \otimes 35$, whereas $20 \rightarrow 70 \otimes 35$ is allowed, decay to $N\pi$ will be allowed only through the **70** admixtures in the nucleon. This makes the $P_{11}(1710)$ and $P_{13}(1720)$ extremely interesting as not only are their total widths significantly less than the $P_{11}(1440)$ but the dominant modes for $P_{11}(1710)$ and $P_{13}(1720)$ are to $N\pi\pi$, which allows a possible cascade decay of $20 \rightarrow N^*(70)\pi \rightarrow N\pi\pi$.

ii) The Λ selection rule is useful for classifying the P_{11} and P_{13} in either $[56,^2 8]$ and $[70,^2 8]$, or $[70,^4 8]$ and $[20,^2 8]$, by looking at their decays into $K\Lambda$ and/or $K^*\Lambda$. The $[70,^4 8]$ decays to $K\Lambda$ will be suppressed relative to $K\Sigma$ for both charged and neutral N^* .

iii) The Moorhouse selection rule can distinguish $[70,^4 8]$ and $[20,^2 8]$ since the $[70,^4 8]$ will be suppressed in γp but sizeable in γn , while the $[20,^2 8]$ will be suppressed in both.

iv) $J/\psi \rightarrow \bar{p} + N^*$ is a further probe of N^* assignments, which accesses **56** in leading order and **70** via mixing while **20** is forbidden. Hence for example $J/\psi \rightarrow \bar{p} + (P_{11} : P_{13})$ probes the **56** and **70** content of these states. Combined with our selection rule this identifies $J/\psi \rightarrow \bar{p} + K\Lambda$ as a channel that selects the **56** content of the P_{11} and P_{13} .

In summary, with interest in N^* with masses above 2 GeV coming into focus at Jefferson Laboratory and accessible at BEPC with high statistic $J/\psi \rightarrow \bar{p} + N^*$, the Λ selection rule should be useful for classifying baryon resonances and interpreting $\gamma N \rightarrow K\Lambda$, $K^*\Lambda$, $K\Sigma$ and $K^*\Sigma$. A coherent study of these channels may provide evidence on the dynamics of diquark correlations and the presence of **20**-plets, which have hitherto been largely ignored.

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